3. E. I. Shemyakin, "Anisotropy of the plastic state," in: Numerical Methods of the Mechanics of Continuous Media [in Russian], 4, No. 4, Novosibirsk (1973).
4. A. M. Kovrizhnykh, "On a modification of the description of the incomplete and complete plasticity states," in: Dynamics of Continuous Media [in Russian], No. 34, Inst. Gidrodinamiki Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1978).
5. W. T. Koiter, "Stress-strain relations uniqueness and variational theorems for elastic-plastic materials with a singular yield surface," Appl. Math., No. 11 (1953).
6. J. L. Sanders, Jr., "Plastic stress-strain relations based on linear loading functions," Proc. Second US Nat. Congr. Appl. Mech., Ann Arbor (1954).
7. V. M. Zhigalkin, N. S. Adigamov, and A. M. Kovrizhnykh, "Investigation of the dependences between stresses and strains in the theory of plasticity of an anisotropically hardening medium," in: Stress Measurement in a Mountain Rock Massif. Materials of the Fourth All-Union Seminar [in Russian], part 1, Izd. Inst. Gornogo Dela Sib. Otd. Akad. Nauk SSSR, Novosibirsk (1974).
8. A. M. Kovrizhnykh, "Taking account of materials of different shear strength beyond the elastic limit," Fiz.-Tekh. Prol. Razrab. Polezn. Iskop., No. 6 (1978).
9. B. Budiansky, N. F. Dow, R. W. Peters, and R. P. Shepherd, "Experimental studies of polyaxial stress-strain laws of plasticity," in: Proc. First US Nat. Cong. Applied Mechanics, Chicago (1951).

STATIONARY CONFIGURATION OF FIBERS FORMED UNDER NONISOTHERMAL CONDITIONS

A. L. Yarin

UDC 532.222 + 681.7.068.4

One of the important problems of chemical technology is fiber molding. Nevertheless, the substantial influence of heat transfer on fiber characteristics has been investigated insufficiently. The first step is to obtain stationary solutions. The stationary fiber configurations are computed numerically in [1]. In this paper analytic solutions of the stationary problem are obtained under the assumption of large activation energy of the viscous flow.

We shall consider the melt to be shaped to be a Newtonian fluid with viscosity dependent on the temperature according to the Arrhenius law. Such high values of the viscosity correspond to sufficiently low temperatures that flow practically ceases and the material solidifies. This approximation corresponds best to the behavior of melted glass [2, 3].

Let us examine the two most widespread technological processes: 1) drawing a fiber from a cylindrical glass blank heated to high temperature (Fig. 1a), and 2) drawing through a spinneret hole from a tank containing the melt (Fig. 1b). In both cases the fiber being drawn cools and solidifies during motion in the air. In the situations under consideration we shall consider the material to advance at a constant given velocity V_0. At the end of the shaping section, the fiber is incident on a receiving unit (bobbin) giving a certain value of the longitudinal velocity. We shall conduct the description within the framework of quasi-one-dimensional equations of continuity, momentum [4, 5], and heat propagation by assuming the flow to change sufficiently slowly along the fiber:

$$\begin{gathered}\partial f/\partial t + \partial fV/\partial x = 0,\ f = \pi a^2,\\ \rho f(\partial V/\partial t + V\partial V/\partial x) = \partial P/\partial x,\ P = 3\mu f\partial V/\partial x,\ \mu = \mu_0 \exp(U/RT),\\ \rho fc\left(\frac{\partial T}{\partial t} + V\frac{\partial T}{\partial x}\right) = \frac{\partial}{\partial x}\left(\lambda f\frac{\partial T}{\partial x}\right) - 2\pi a q_w.\end{gathered} \tag{1}$$

Here t is the time, x is the coordinate measured along the fiber axis, f is the area of the fiber section (it is considered that it has a circular section of radius a), V is the magnitude of the axial velocity in the fiber, T is the temperature, ρ, μ, c, λ are the density, viscosity, specific heat, and heat conduction of the melt, P is the magnitude of the axial force in the fiber section, μ_0 and U are the preexponential factor and the activation energy

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 139-144, November-December, 1982. Original article submitted October 26, 1981.

0021-8944/82/2306-0865$07.50

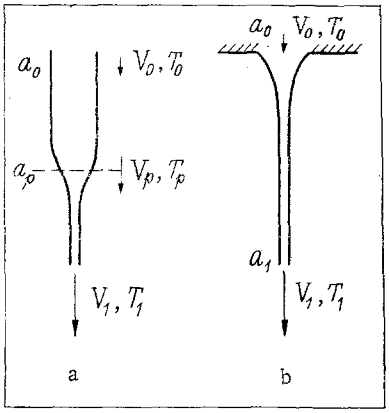

Fig. 1

of the viscous flow, R is the gas constant, and q_W is the heat flux in the direction of the external normal to the side surface of the fiber.

We neglect gravity, friction drag in air, and surface tension of the melt. We consider the viscous forces so large that by comparison the inertial effects (the left side of the second equation in (1)) is negligible. Moreover, we will neglect longitudinal conductive heat transfer in the fiber. The assumptions made are physically justified and repeatedly applied earlier [6, 7]. We note that in the situation when the fiber is drawn continuously from the blank, (1) is used also to describe the flow development in the blank. The system (1) acquires the following form in the stationary case under consideration

$$fV = Q,\ dP/dx = 0,\ \rho c Q dT/dx = -2\pi a q_w, \tag{2}$$

where Q is the volume flux determined by the boundary conditions of the problem, $Q = \pi a_1^2 V_1$ (a_1 and V_1 are the fiber radius and velocity going to the receiving unit). Taking account of the continuity equation, we convert the momentum and heat distribution equations (2) to the form

$$\frac{1}{2V}\frac{dV}{dx} = C\exp(-U/RT),\ \frac{dT}{dx} = \frac{A_w}{\sqrt{V}}, \tag{3}$$

where C is a constant of integration, and $A_W = -(2q_W/\rho c)\sqrt{\pi/Q}$. (Later we shall denote the heat flux in the area of the blank by $q_W = q < 0$ and $A_W = A$, respectively, while in the drawing stage where cooling occurs $q_W = q' > 0$ and, respectively, $A_W = A'$.)

Introducing the function $\varphi = \sqrt{V}$, we obtain the following system from (3)

$$d\varphi/dx = C\varphi\exp(-U/RT),\ dT/dx = A_w/\varphi. \tag{4}$$

Selecting T as variable, we have

$$d\varphi/dT = C\varphi^2/A_w \exp(-U/RT). \tag{5}$$

We shall consider that heating of the glass from an initial temperature T_0 to a temperature T_p occurs in a section of a certain, still not determined, length l in the blank while considering molding from the blank, and the fiber temperature drops to T_1 on a section of still undetermined length l_1 ahead of the receiving unit in the drawing stage. In general, l and l_1 can be given, but the quantities T_p and T_1 should be determined, but since dependences of the lengths l and l_1 on the corresponding temperatures will be obtained as a result of solving the problem, then the problem formulation taken is simply more convenient.

Using the notation $\varphi_0 = \sqrt{V_0}$, and integration (5), we obtain

$$\int_{\varphi_0}^{\varphi}\frac{d\varphi}{\varphi^2} = C\int_{T_0}^{T} A^{-1}\exp(-U/R\tilde{T})\,d\tilde{T}. \tag{6}$$

For sufficiently high values of the activation energy U (as really occur [2]), the integrand $A^{-1}\exp(-U/R\tilde{T})$ in (6) changes, because of the substantial nonlinearity of the Arrhenius law, only in a narrow boundary layer near T in the interval $T_0 \quad \tilde{T} \leqslant T$ independently of the nature of the variation of A^{-1} with $\tilde{T}$; the heat flux is a power-law function of $\tilde{T}$. This permits utilization of the Laplace method [8], and obtaining an asymptotic expression for the integral in the form

$$\int_{T_0}^{T} A^{-1} \exp(-U/R\tilde{T})\, d\tilde{T} = \frac{A^{-1}(T) RT^2}{U} \exp(-U/RT) \simeq \frac{A^{-1}(T_p) RT_p^2}{U} \exp(-U/RT_p) \exp\left[\frac{U}{RT_p^2}(T - T_p)\right].$$

The approximate second equality is obtained here by using the D. A. Frank-Kamenetskii method of decomposition [9]. Consequently, a good approximation is obtained when T is close to T_p; in the opposite case both the function itself and its approximation from [9] are almost zero, and therefore, the approximation is again satisfactory.

Therefore, by understanding A to be the value of $A(T_p)$ we obtain the relationship between the velocity and the temperature in the blank in the form

$$\frac{1}{\varphi} = \frac{1}{\varphi_0} - \frac{C}{A} \frac{\exp(-U/RT_p)}{U/RT_p^2} \exp\left[\frac{U}{RT_p^2}(T - T_p)\right]. \tag{7}$$

Substituting (7) into the second equation in (4), and integrating with the boundary condition $x = 0$, $T = T_0$ taken into account, we find the dependence of the temperature in the blank on the longitudinal coordinate

$$\frac{(T - T_0)\varphi_0}{A} - \frac{\varphi_0}{AU/RT_p^2} \ln\left\{1 - \frac{C \exp(-U/RT_p)\varphi_0}{AU/RT_p^2} \exp\left[\frac{U}{RT_p^2}(T - T_p)\right]\right\} = x. \tag{8}$$

In place of the unknown constant C we introduce the magnitude of the velocity at the end of the blank (at the end of the heating domain) V_p (and $\varphi_p = \sqrt{V_p}$), which is also still unknown. Taking account of (7) we obtain

$$C = \left(\frac{1}{\varphi_0} - \frac{1}{\varphi_p}\right)(AU/RT_p^2) \exp(U/RT_p). \tag{9}$$

Moreover, by using (8) we obtain a dependence of the length of the heating zone l on T_p:

$$l = \frac{(T_p - T_0)\varphi_0}{A} - \frac{\varphi_0}{AU/RT_p^2} \ln \frac{\varphi_0}{\varphi_p}. \tag{10}$$

The relation between the length l and T_p is thereby set.

Going over to dimensionless variables in (7) and (8) and using the continuity equation, by taking account of (8) and (9) we obtain a parametric dependence of the radius of the blank in the heating zone on the longitudinal coordinate

$$\begin{gathered} \bar{a} = \sqrt{E} - (\sqrt{E} - \bar{a}_p) \exp[\theta(\bar{T} - 1)], \\ \bar{x} = [\bar{T} - \bar{T}_0 - \theta^{-1} \ln\{1 - (1 - \bar{a}_p/\sqrt{E}) \exp[\theta(\bar{T} - 1)]\}]L^*/L. \end{gathered} \tag{11}$$

Here the radius is referred to a_1, the temperature to T_p, x to the total length of the fiber being molded $L = l + l_1$; $\bar{a}_p$ is the dimensionless value of the radius at the end of the heating zone (still unknown), and E is the multiplicity of the drawing which equals $V_1/V_0 = (a_0/a_1)^2$. In addition

$$\begin{gathered} L^* = -\rho c T_p Q/2q\pi a_0, \ \theta = U/RT_p, \\ l = L^*[1 - \bar{T}_0 + \theta^{-1} \ln(\sqrt{E}/\bar{a}_p)] \ (q = q(T_p) < 0). \end{gathered}$$

The unknown quantity $\bar{a}_p$ will be determined after having solved the problem of behavior of the fiber in the drawing stage when it cools ($q' > 0$). This latter solution should, in combination with (11), assure continuity of the distributions of the radius, the velocity, and the force in the fiber section along its length, by which $\bar{a}_p$ is indeed determined. In the drawing stage (5) is integrated in the form

$$\int_{\varphi_p}^{\varphi} \frac{d\varphi}{\varphi^2} = C \int_{T_p}^{T} (A')^{-1} \exp(-U/R\tilde{T})\, d\tilde{T}. \tag{12}$$

Here, as before, $\varphi_p = \sqrt{V_p}$ and V_p is the unknown velocity at the end of the heating zone, i.e., in the initial section of the drawing part.

Using the Laplace method and decomposition [9], we find

$$\frac{1}{\varphi} = \frac{1}{\varphi_p} - \frac{C}{A'}\frac{\exp(-U/RT_p)}{U/RT_p^2}\left\{\exp\left[\frac{U}{RT_p^2}(T-T_p)\right]-1\right\}, \quad (13)$$

$$A' = A'(T_p).$$

Taking account of the boundary condition on the receiving unit $\varphi = \varphi_1 = \sqrt{V_1}$ for $T = T_1$ we obtain from (13)

$$C = \left(\frac{1}{\varphi_1} - \frac{1}{\varphi_p}\right)(A'U/RT_p^2)\exp(U/RT_p). \quad (14)$$

Continuity of the force distribution along the fiber is assured by the equality of the expressions (9) and (14): the force in the section is $P = 6\mu_0 QC$. Therefore, $(1/\varphi_0 - 1/\varphi_p)A = (1/\varphi_1 - 1/\varphi_p)A'$. Finding φ_p from this latter equation and using the constancy of the volume flux along the fiber, we obtain the following expression for $\bar{a}_p$

$$\bar{a}_p = (\sqrt{E} - A'/A)/(1 - A'/A). \quad (15)$$

By using (15), the equations (11) permit determination of the profile of the blank in the heating zone. Let us note that $A'/A < 0$.

Using the velocity-temperature relationship described in the fiber cooling and drawing zones by (13) and (14), and also the continuity equation, we can obtain a dependence of the radius on the temperature. Then by integrating the second equation in (4), we obtain the temperature distribution along this zone:

$$\bar{a} = \bar{a}_p\left[1 - \frac{1-\sqrt{E}}{\sqrt{E}-A'/A}\{\exp[\theta(\bar{T}-1)]-1\}\right],$$

$$\bar{x} = \frac{l}{L} + \frac{\sqrt{E}L^*}{L}\frac{A}{A'}\left[\bar{T} - 1 - \theta^{-1}\ln\left\{\frac{1-A'/A}{\sqrt{E}-A'/A} - \frac{1-\sqrt{E}}{\sqrt{E}-A'/A}\exp[\theta(\bar{T}-1)]\right\}\right]. \quad (16)$$

As before, the upper bar denotes dimensionless quantities.

Setting $\bar{T} = \bar{T}_1$ in the last equation of (16), we calculate the total length of the fiber from the section in which heating of the blank started to the section in which the fiber is cooled to $\bar{T} = \bar{T}_1$:

$$L = l + l_1 = l + \sqrt{E}L^*\frac{A}{A'}\left[\bar{T}_1 - 1 - \theta^{-1}\ln\left(\frac{1-A'/A}{\sqrt{E}-A'/A}\right)\right].$$

The connection between the length l_1 and T_1 is thereby set. The expression for the longitudinal force in a fiber being drawn from a blank has the form

$$P = \{[12\pi\mu_0 a_0(-q)\theta\exp(\theta)]/(\rho c T_p)\}\,(1 - \bar{a}_p/\sqrt{E}).$$

Determination of the stationary configuration of a fiber being drawn from a spinner is perfectly analogous to the consideration made above for the cooling zone of a fiber being drawn from a blank. The result has the form

$$\bar{a} = \sqrt{E} - (1-\sqrt{E})\{\exp[\theta(\bar{T}-1)]-1\},$$

$$\bar{x} = \frac{\bar{T} - 1 - \theta^{-1}\ln\{E^{-1/2} - (E^{-1/2}-1)\exp[\theta(\bar{T}-1)]\}}{\bar{T}_1 - 1 + (2\theta)^{-1}\ln E}. \quad (17)$$

Here, as before, the radius is referred to a_1, the temperature to the initial value T_0, and the scale for the longitudinal coordinate is

$$L_1 = -\frac{\rho c T_p V_1 a_1}{2q'}\left(\bar{T}_1 - 1 + \frac{\theta^{-1}}{2}\ln E\right) \quad (q' = q'(T_0) > 0) \quad (18)$$

the length in which the fiber temperature drops because of heat elimination from an initial value T_0 known in advance on the spinner exit to the final value T_1 on the receiving unit. Conversely, the value L_1 can be considered given, and T_1 can be determined from (18).

It has therefore been obtained that the stationary configuration of a fiber being drawn from a blank is described by (11), (15), (16) while a fiber being drawn from a spinner is described by (17). By passing to the limit $\theta \to 0$ the results obtained do not permit arriving at the known isothermal solution $\bar{a} = (\sqrt{E})^{1-\bar{x}}$ [6] (drawn from a spinner). This is natural since all the results obtained here correspond to the asymptotic $\theta \gg 1$.

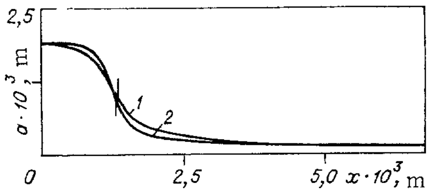

Fig. 2

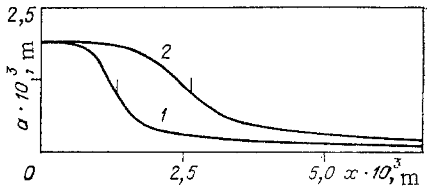

Fig. 3

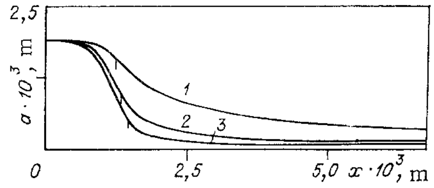

Fig. 4

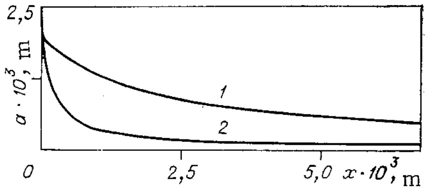

Fig. 5

Let us illustrate the results obtained. In the case of drawing fiberglass from a heated blank, it was assumed in the computation that the radius of the blank is $a_0 = 0.19 \cdot 10^{-2}$ m, its temperature is $T_0 = 300$°K, the temperature to which the blank is heated is $T_p = 1873$°K, the radius of the fiber at the receiving unit is $a_1 = 6.25 \cdot 10^{-5}$ m, the velocity is $V_1 = 0.3$ m/sec, the temperature is $T_1 = 300$°K.

The glass density is $\rho = 2.2 \cdot 10^3$ kg/m³, its specific heat is $c = 1.043 \cdot 10^3$ J/(kg·deg). It was assumed that $q(T_p) = -92.7 \cdot 10^4 \varepsilon$ kg/(m²·sec), where ε is a dimensional factor with the meaning of the emissivity of the glass.

Configurations of the fibers being drawn, which have different values of the dimensionless activation energy θ, are represented in Fig. 2, where $\varepsilon = 1$, $q'/q = A'/A = -1$, the value $\theta = 7$ corresponds to curve 1, $\theta = 10$ to 2, the vertical bars display the position of the heating zone boundaries. It is seen that as θ increases, the representation of quasi-one-dimensionality of the flow in the heating region has less and less of a foundation. In the example presented, the multiplicity of the drawing is quite large (E = 924); as E decreases the quasi-one-dimensional nature of the flow in the heating domain is not spoiled for values of θ substantially greater than 10.

Diminution of the heat flux supplied and eliminated for fixed values of θ and the ratio $A'/A(q'/q)$ results in a smoother passage from the heating of the drawing zone, as the data in Fig. 3 indicate, where $\theta = 7$, $A'/A = -1$, $\varepsilon = 1$, corresponds to curve 1, and $\varepsilon = 0.5$ to curve 2.

The influence of the relative intensity of heat flux elimination and delivery on the configuration of the fiber being molded is illustrated in Fig. 4, where $\theta = 7$, $\varepsilon = 1$, $A'/A = -0.2$ corresponds to curve 1, $A'/A = -1$ to curve 2, $A'/A = -3$ to 3. An increase in the heat elimination intensity for a fixed heat flux to the blank results in a noticeable acceleration of fiber molding. The total length of the fiber being molded diminishes abruptly with the increase in heat elimination when the temperature T_1 is given. However, intensification of the heat elimination also results in noticeable growth of the longitudinal force in the fiber, which can cause its rupture. Computed profiles of fibers molded by drawing from a spinner with radius $0.19 \cdot 10^{-2}$ m are shown in Fig. 5. The magnitudes of the heat fluxes $q' \cdot (T_0) = 92.7 \cdot 10^3$ J/(m²·sec) (curve 1) and $q'(T_0) = 92.7 \cdot 10^4$ J/(m²·sec) (curve 2), $T_0 = 1873$°K, and the values of the remaining fiber parameters are the same as in the computations for drawing from blanks.

LITERATURE CITED

1. U. C. Paek and R. B. Runk, "Physical behavior of the neck-down region during furnace drawing of silica fibers," J. Appl. Phys., 49, No. 8 (1978).
2. R. H. Doremus, Glass Science, Wiley, New York (1973).
3. A. Zyabitskii, Theoretical Principles of Fiber Molding [in Russian], Khimiya, Moscow (1979).
4. V. M. Entov and A. L. Yarin, "Dynamics equations of a jet of dropping fluid," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 5 (1980).
5. V. M. Entov, V. I. Kordonskii, et al., "Investigation of jet dissociation of rheologi-

cally complex fluids," Prikl. Mekh. Tekh. Fiz., No. 3 (1980).
6. M. A. Matovich and J. R. A. Pearson, "Spinning a molten threadline. Steady-state viscous flows," Ind. Eng. Chem. Fund. 8, No. 3 (1969).
7. S. Kase, "Studies on melt spinning. IV. On the stability of melt spinning," J. Appl. Polymer Sci., 18, No. 11 (1974).
8. M. A. Evgrafov, Asymptotic Estimates and Entire Functions [in Russian], Nauka, Moscow (1979).
9. D. A. Frank-Kamenetskii, Diffusion and Heat Transfer in Chemical Kinetics [in Russian], Nauka, Moscow (1967).

STABILITY OF THE PLANE WAVE FRONT OF FLUID EVAPORATION

E. B. Levchenko and A. L. Chernyakov

UDC 532.70 + 535.211

An evaporation wave is propagated in the bulk of a substance subjected to a powerful radiation flux in a condensed medium. In those cases when the domain thickness in front of the wave front heated because of heat conduction is small compared with the characteristic dimensions of the system under consideration, the realization of a quasistationary regime for which the velocity of wave front motion is determined by the instantaneous value of the energy flux density absorbed in the medium, is generally possible. In fact, the process of material rupture under sufficiently large energy flux intensities (for $Q > 10^5$–10^6 W/cm^2 for metals) is accompanied, as a rule, by different nonstationary phenomena such as self-oscillations in the gas flow, ejection of substance in the form of drops, etc. [1], which apparently indicates instability of the quasistationary evaporation mode.

In this paper the stability of the plane fluid evaporation wave front considered as the surface of discontinuity of the thermodynamic functions of the substance, is investigated. An analogous problem in the theory of slow combustion was investigated by Landau [2], who also discovered the instability mechanism of a plane chemical reaction wave associated with the development of vortical disturbances in the flux of combustion products. In application to the process of substance evaporation by powerful radiation flux, the mentioned instability mechanism turns out to be decisive for the development of fluctuations of a front with wavelengths commensurate to the diameter of the radiation focusing spot. A substantial feature of the evaporation process, because of which results obtained in the theory of slow combustion [2, 3] are not directly applicable to the latter, is the high velocity of vapor escape, which is commensurate with the speed of sound in a gas. Taking account of the vapor compressibility, which is necessary in this case, results in a change in both the conditions of origination and the nature of the development of the instability of the plane fluid evaporation wave front.

Let us select a reference system in which the plane evaporation wave front is at rest, and we direct the Cartesian z axis along the normal to the front so that the domain $z < 0$ is filled with fluid and $z > 0$ with vapor. In this coordinate system the temperature profile is stationary and has the following form in the absence of radiation absorption in the vapor during surface evaporation:

$$T_0(z) = \begin{cases} T_{0l}(z), & z < 0, \\ T_{0g}(z) = \text{const}, & z > 0, \end{cases}$$

$$T_{0l} = T_{0S} \exp\left(\frac{v_l z}{\chi_l}\right) + \frac{Q}{\varkappa_l} \frac{e^{(v_l z/\chi_l)} - e^{\mu z}}{\mu - v_l/\chi_l},$$

where Q is the energy flux density, μ is the coefficient of radiation absorption, $\varkappa_l = \rho_l c_l \chi_l$ is the heat conduction, and c_l, ρ_l is the specific heat and density of the fluid. The surface temperature T_{0S} and the flow velocity v_l are determined from the energy conservation law

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 144-150, November-December, 1982. Original article submitted February 10, 1982.

 0021-8944/82/2306-0870$07.50